

Notes for the Teacher

Students decompose two-digit multiplication problems into tens and ones by manipulating a dynamic array of base-ten blocks (for example, $27 \times 39 = (20 + 7) \times (30 + 9)$). Students compute the partial products in their array and then add them together to find the overall product.

Objectives:

- Students will find products of two-digit numbers by decomposing each number into tens and ones.
- Students will use a rectangular array model composed of base-ten blocks to explore two-digit multiplication problems.
- Students will understand the relationship between a multiplication problem represented in numerical form and the same problem represented with an array.

Common Core Mathematical Practices: (1) Make sense of problems and persevere in solving them; (2) Reason abstractly and quantitatively; (3) Construct viable arguments and critique the reasoning of others; (5) Use appropriate tools strategically; (7) Look for and make use of structure.

Common Core State Content Standards: 4.NBT5

Grade Range: Grades 3–5

Preparation:

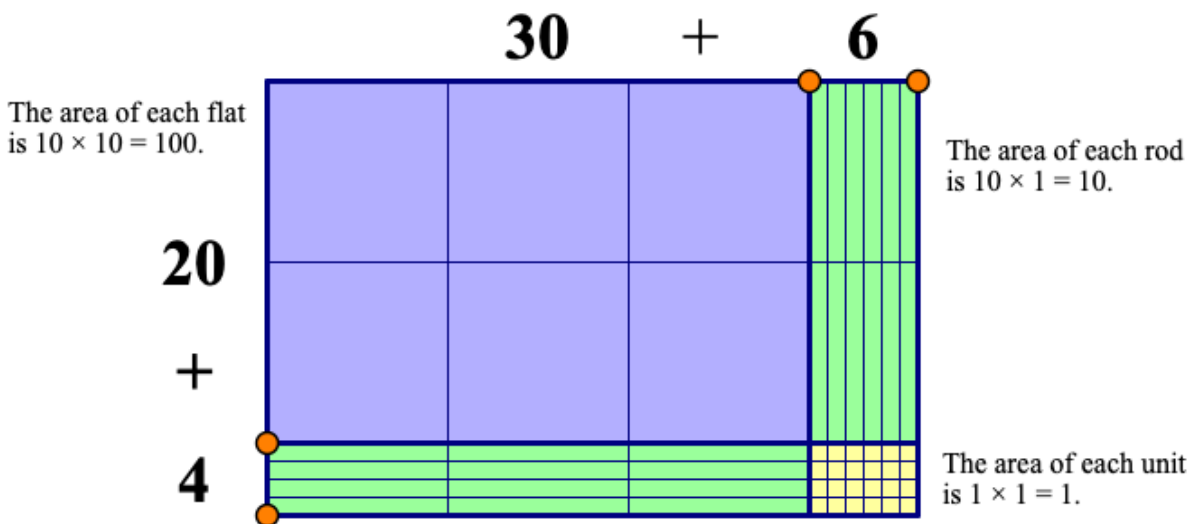
This activity is particularly effective with students who have already used physical Base Ten blocks (available from a number of sources).

Before class, open <http://sineofthetimes.org/2018/basetenblocks2/>, download the worksheet, and make copies for your students to use.

Introduce:

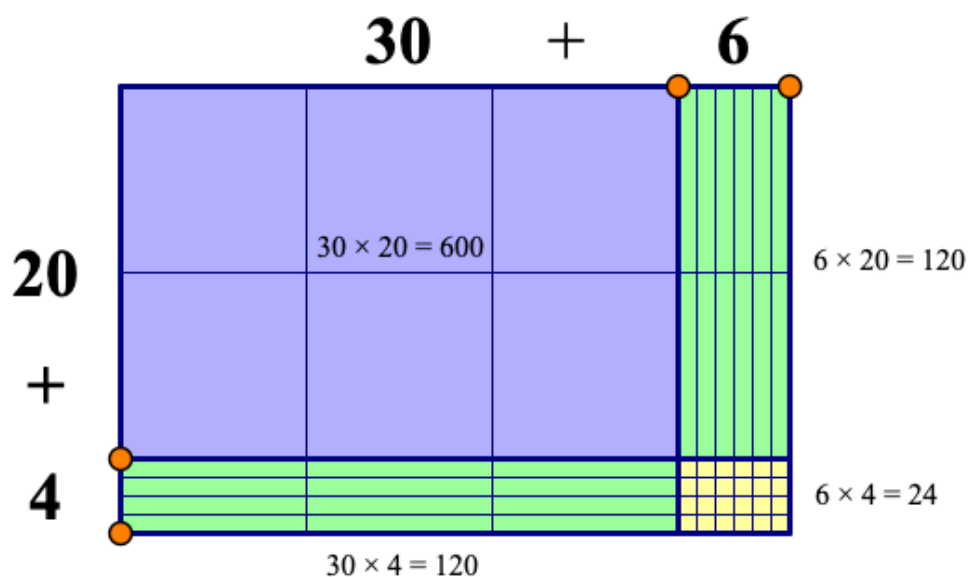
Open <http://sineofthetimes.org/2018/basetenblocks2/> and distribute the worksheet. Use a projector to show page 1. Ask, “What multiplication problem is represented by this array?” The array shows 36×24 represented as $(30 + 6) \times (20 + 4)$ with base-ten blocks. The flats (large purple squares) represent hundreds, the rods (long green rectangles) represent tens, and the small yellow squares represent units. If you have base-ten blocks, show them to the class so that they can relate the physical blocks to their virtual counterparts.

Ask students to find the dimensions and area of each block type. If you're using a whiteboard, write this information directly onto the model:



Press the *Show Units* button to show how each flat really does contain 100 units, and each rod really does contain 10 units. Press *Hide Units* to restore the original view.

There are four color-coded arrays within the overall array: a purple array of flats, two green arrays of rods, and a yellow array of units. Ask students to state the dimensions and areas of the four arrays. Students may describe the smaller areas in terms of base-ten blocks. For example, the area of the flats is 6 hundreds. Alternatively, students may describe the area as $30 \times 20 = 600$ or $100 \times 6 = 600$. Solicit different ways to correctly describe the areas; it will help you understand students' thinking and expose students to other ways of thinking about the areas.



Introduce the term *partial product* to describe each product represented by a smaller array. Explain that we say *partial* to mean that each array represents just part of the overall area. Students should also understand that to find the total product, they need to add all of the partial products together.

Write the partial products on the board as students state them. There are four partial products, one for each smaller array. When summed, their area represents the product 36×24 :

$$600 + 120 + 120 + 24 = 864$$

Press the *Product On/Off* button to check.

Ask a student to drag the orange points to create a new multiplication problem. If the array becomes too large for the window size, drag the point on the vertical 1's slider to adjust the sizes of the blocks, or drag the 10's slider for more precise control. Have volunteers take turns creating and solving several problems until students understand how to use and interpret the model.

Explore:

Assign students to partners and send them in pairs to the computers. Have students open **Two-Digit Multiplication Part One--Using Base Ten Blocks.gsp** and go to page 1. Ask students to use the model to find the products of two-digit multiplication problems. Make sure students understand how to record the results on the worksheet. Explain that students should sketch how the array is split, find the partial products, and then calculate their sum.

Indicate the three buttons on the sketch and explain what each does:

- *Grid Lines On/Off*: Press this button to show/hide grids lines outlining the individual base-ten blocks.
- *Partial Products On/Off*: Press this button to show the partial products inside the arrays.
- *Product On/Off*: Press this button to see an expression for the sum of the partial products. Student can use this button to check their work.

As you circulate, observe students as they work. Do they need to see the grid lines outlining the base-ten blocks? Can students find the partial products easily? How do students represent the multiplication problems? Are they decomposing the factors of the product by place value, separating the tens and ones?

For students who are comfortable with the model, consider asking them to use the related model on page 2 of the sketch. This model is the same as the one on page 1, but all the grid lines have been hidden.

Discuss:

Call students together to discuss the worksheet. Open **Two-Digit Multiplication Part One--Using Base Ten Blocks.gsp** and go to page 1 or 2. Ask volunteers to model the solutions. Ask students questions such as the following as they share their solutions:

- *How does your array model represent the multiplication problem?*
- *Why is it helpful to split each number into tens and ones?*
- *What do the partial products show?*
- *Why did you add the partial products together?*

Answers:

$$33 \times 26 = 858$$

$$28 \times 52 = 1,456$$

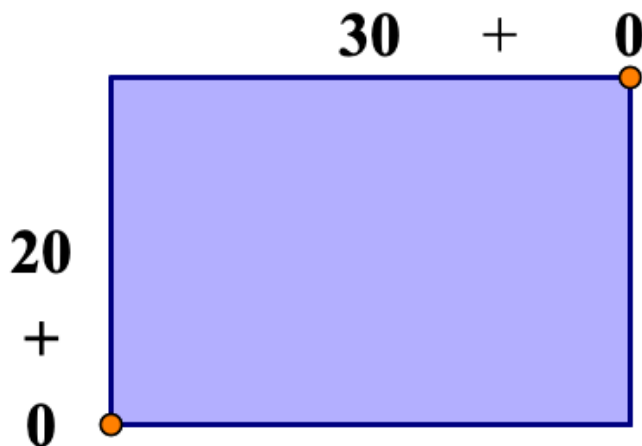
$$43 \times 20 = 860$$

$$45 \times 45 = 2,025$$

$$62 \times 79 = 4,898$$

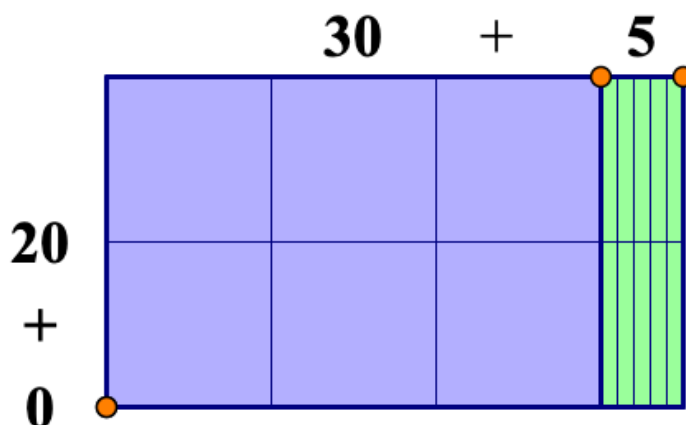
Explore More:

As an extension, show 20×30 using the representation below:



Drag the top-right point to show 20×31 , 20×32 , 20×33 , and so on. Pause each time you drag and ask, “What happened to the area of the entire array?” (The area of the purple array stays the same but the area of the green array increases by 20 each time.) “Explain what is happening.” Students may reply, “Two more tens are being added for each unit added to the length,” or “The area increases by 20 for each unit added because that’s the length of one side of the rectangle.” If needed, press the *Grid Lines On/Off* button to show the blocks so that students see what is happening. Ask, “How can we compute 20×38 ?” Students may reply that the product is equal to 20×30 (600) plus 20×8 (160).

Now show 20×35 using the representation below:



Drag the array to show 21×35 , 22×35 , 23×35 , and so on. Pause each time you drag and ask, “What happens to the area of the entire array?” (The area of the green array increases by 30 each time and the area of the yellow array increases by 5 for each unit added to the width.) “Explain what is happening.” Students may state, “Three tens and 5 ones are being added for each unit added to the width,” or “The combined area is 35, and we are adding another 35 for each unit added to the width.”

Student responses will give you insight into their understanding of the model.

Related Activities:

- *Two-Digit Multiplication—Using an Array Model*
- *Two-Digit Multiplication, Part Two—Using Negative Units*
- *Bunny Times*

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