Circles, Angles, and Arcs

Here are four versions of the same activity, designed for students with different familiarity with Sketchpad and with different needs for specific support in the course of doing the activity. The activities are in order of increasing challenge, with the first page of each labeled 1, 2, 3, or 4.

This is an experimental collection, for the purpose of getting feedback and ideas about how to provide an activity such as this in a way that supports the diversity of students' experiences and needs, while still supporting a high level of cognitive demand even for students with a stronger Sketchpad background.

I'd appreciate comments, from students and teachers alike, on the value and feasibility of this approach of providing several different versions of an activity.

If you're a student, which of the four versions is most appropriate for you? How can the format encourage you to try a higher-level version first, so that you're really challenged? How much help is it to know that you can drop back to a lower-level version if you need the extra support?

If you're a teacher, which of these four variations would be useful for your range of students? How can you imagine presenting this activity in a way that encourages students to try the more challenging level first, only dropping back to a less-challenging version if necessary? Does the availability of all four versions make it too attractive for students to take the easy way out?

I will be grateful for any and all comments you care to send me.

Thanks,

Scott
Scott Steketee (stek@kcptech.com)

PS. One way of categorizing the level of cognitive demand of a task like this is the Mathematical Task Framework, described in (Stein 2009). For a discussion of how to maintain cognitive demand in actual classroom practice, see (Henningsen 2003).

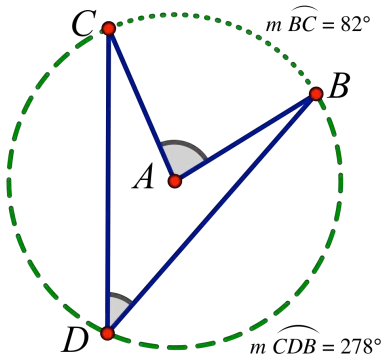
References:

Stein, M. K., Smith, Henningsen, and Silver, 2009. Implementing Standards-Based Mathematics Instruction: A Casebook for Teachers, Teachers College Press, 2009.
http://store.tcpress.com/0807749575.shtml

Henningsen, M. and Stein, and Silver, 2002. "Supporting Students' High-Level Thinking, Reasoning, And Communication In Mathematics" from Lessons Learned from Research, NCTM, 2002.
http://www.nctm.org/Catalog/product.aspx?id=12300

Contents

Circles, Angles, and Arcs Name:



Terminology of circles, angles, and arcs

A is the center of $\odot AB$.

$\angle BAC$ is a *central* angle of the circle, because its vertex is at the center.

$\angle BDC$ is an *inscribed* angle, because its vertex is on the circle and both its sides intersect the circle.

$\overset{\frown}{BC}$ is a *minor* arc, because it is less than a semi-circle (*semi* means half).

$m\overset{\frown}{BC}$ measures the *arc angle* of $\overset{\frown}{BC}$.

$\overset{\frown}{CDB}$ is a *major* arc, because it is more than a semi-circle.

You can use the tips listed below to help you with the activity *Circles, Angles, and Arcs*. To see any of these tips from Sketchpad, choose **Help | Using Sketchpad | Sketchpad Tips.** (All the tips come in both comic and video format.)

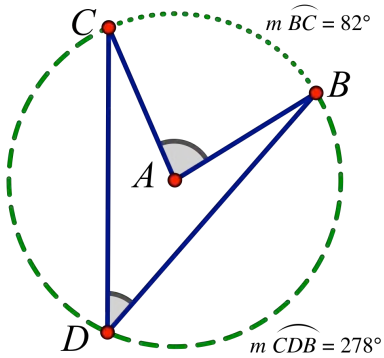
TIPS FOR THE DETAILED STEPS

Step 1	Tools	Using the Compass Tool
Step 2, 3	Tools	Using the Straightedge Tool
Step 4	Tools	Using the Marker Tool Tools	Working with Angle Markers
Step 5	Measure	Measuring Angles
Step 6	Construct	Constructing Arcs
Step 7	Tools	The Drag Test
Step 8	Measure	Measuring Arc Angles and Arc Lengths
Step 10	Tools	Using the Point Tool

TIPS FOR PRESENTING YOUR WORK

Explaining	Tools	Using the Text Tool Tools	Using Hot Text
Presenting	Edit	Making Hide/Show Buttons Edit	Making Movement Buttons Edit	Making Presentation Buttons

Circles, Angles, and Arcs Name:



Terminology of circles, angles, and arcs

A is the center of $\odot AB$.

$\angle BAC$ is a *central* angle of the circle, because its vertex is at the center.

$\angle BDC$ is an *inscribed* angle, because its vertex is on the circle and both its sides intersect the circle.

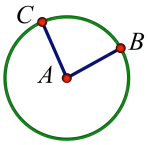
$\overset{\frown}{BC}$ is a *minor* arc, because it is less than a semi-circle (*semi* means half).

$m\overset{\frown}{BC}$ measures the *arc angle* of $\overset{\frown}{BC}$.

$\overset{\frown}{CDB}$ is a *major* arc, because it is more than a semi-circle.

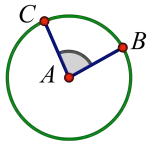
In this activity, you'll investigate relationships among central angles, inscribed angles, and the arcs they intercept.

CONSTRUCT AND CONJECTURE



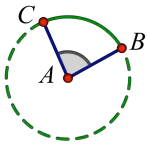
1. Construct circle AB using the **Compass** tool.
2. Construct segment AB using the **Segment** tool.
3. To create central angle $\angle BAC$, construct segment AC. Be sure to attach point C to the circle.

Drag the **Marker** tool from point A to the inside of the angle.



$m\angle BAC = 84°$

4. Draw an angle marker in $\angle BAC$ using the **Marker** tool.
5. Measure $\angle BAC$ by selecting the angle marker and choosing **Measure | Angle.**
6. Construct arc BC by first selecting the circle and then the two points in counter-clockwise order. To complete the construction, choose **Construct | Arc on Circle.**
7. Drag C around the circle to see how it controls the arc. Leave point C in a position where the arc is a minor arc.



$m\angle BAC = 84°$

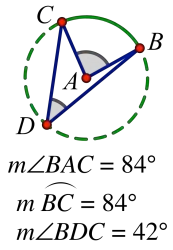
$m\overset{\frown}{BC} = 84°$

8. Measure the angle of arc BC by selecting the arc and choosing **Measure | Arc Angle.**
9. Drag C around the circle again and observe the measurements. Pay attention to the differences when the arc is a minor arc and when it is a major arc.

Q1 Write a conjecture about the measurement of the central angle and the measurement of the minor arc it intercepts.

Q2 Write a conjecture about the measurement of the central angle and the measurement of the major arc.

10. Construct point *D* on the circle using the **Point** tool. Place point *D* on the opposite side of the circle from arc *BC*.
11. Construct segments *DB* and *DC* using the **Segment** tool.
12. Draw an angle marker in $\angle BDC$ using the **Marker** tool.
13. Measure $\angle BDC$ by selecting the angle marker and choosing **Measure | Angle.**



Q3 Write a conjecture about the measurements of an inscribed angle and the arc it intercepts.

14. Drag point *D* back and forth, but not past point *C* or point *B*. As you drag, observe the measurement of $\angle BDC$.

Q4 Write a conjecture about all the inscribed angles that intercept the same arc.

15. Drag point *C* so that the thick arc is as close to being a semicircle as you can make it. Drag *D* and observe the measurement of $\angle BDC$.

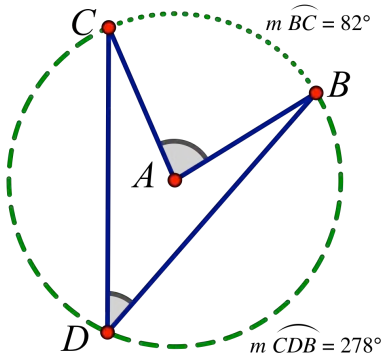
Q5 Write a conjecture about angles inscribed in a semicircle.

EXPLORE MORE

EM1 In a new sketch, construct a circle and an arc on the circle. Measure the circumference of the circle, the arc angle, and the arc length.
Use the circumference and arc angle measurements to calculate an expression equal to the arc length. Explain what you did.

EM2 Use your conjecture in Q5 to come up with a method for constructing a right triangle. Describe your method.

Circles, Angles, and Arcs Name:



Terminology of circles, angles, and arcs

A is the center of $\odot AB$.

$\angle BAC$ is a *central* angle of the circle, because its vertex is at the center.

$\angle BDC$ is an *inscribed* angle, because its vertex is on the circle and both its sides intersect the circle.

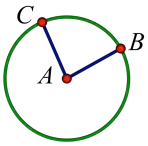
$\overset{\frown}{BC}$ is a *minor* arc, because it is less than a semi-circle (*semi* means half).

$m\overset{\frown}{BC}$ measures the *arc angle* of $\overset{\frown}{BC}$.

$\overset{\frown}{CDB}$ is a *major* arc, because it is more than a semi-circle.

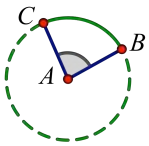
CONSTRUCT AND CONJECTURE

1. Construct circle AB, radius segment AB, and radius segment AC, where C is a point on the circle.



2. Construct the arc on the circle from point B to point C. Drag C around the circle to see how it controls the arc. Position point C so that the arc is a minor arc.

Select the circle and points B and C. Choose **Construct | Arc on Circle.**



3. Draw an angle marker in $\angle BAC$, and measure the angle. Also measure the angle of arc BC.

Use the **Marker** tool to draw the marker. Select the angle marker that you just drew, and choose **Measure | Angle.**

$m\angle BAC = 84°$

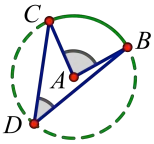
$m\overset{\frown}{BC} = 84°$

→ 4. Drag point C around the circle again and observe the measurements. Pay attention to the differences when the arc is a minor arc and when it is a major arc.

Q1 Write a conjecture about the measurement of the central angle and the measurement of the minor arc it intercepts.

Q2 Write a conjecture about the measurement of the central angle and the measurement of the major arc.

5. Construct and measure the inscribed $\angle BDC$. Drag point C and observe the measurements of the arc angle and $\angle BDC$.



$m\angle BAC = 84°$

$m\overset{\frown}{BC} = 84°$

$m\angle BDC = 42°$

Q3 Write a conjecture about the measurements of an inscribed angle and the arc it intercepts.

6. Drag point D (but not past point C or point B) and observe the measurement of $\angle BDC$.

Q4 Write a conjecture about all the inscribed angles that intercept the same arc.

7. Drag point C so that the thick arc is as close to being a semicircle as you can make it. Drag D and observe the measurement of $\angle BDC$.

Q5 Write a conjecture about angles inscribed in a semicircle.

EXPLORE MORE

EM1 In a new sketch, construct a circle and an arc on the circle. Measure the circumference of the circle, the arc angle, and the arc length. Use the circumference and arc angle measurements to calculate an expression equal to the arc length.
Explain what you did.

EM2 Use your conjecture in Q5 to come up with a method for constructing a right triangle. Describe your method.

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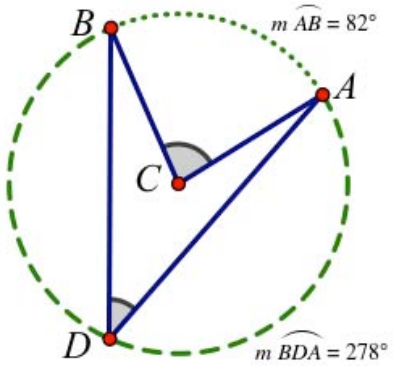

Terminology of circles, arcs, and angles

C is the center of $\odot CA$.

$\angle ACB$ is a *central* angle of the circle, because its vertex is at the center.

$\angle ADB$ is an *inscribed* angle, because its vertex is on the circle and both its sides intersect the circle.

$\overset{\frown}{AB}$ is a *minor* arc, because it is less than a semi-circle.

$m\overset{\frown}{AB}$ measures the *arc angle* of $\overset{\frown}{AB}$.

$\overset{\frown}{BDA}$ is a *major* arc, because it is more than a semi-circle.

FORM AND TEST CONJECTURES

For each question below, start on a blank page and construct the required circle, angle(s), and measurements. Experiment by dragging points while observing the construction and the measurements. Once you've made a conjecture, create three things: (1) a Hot Text caption stating the conjecture, (2) an Animation or Movement button to illustrate the conjecture, and (3) a Hot Text caption explaining why you think the conjecture is true.

Q1 Write a conjecture about the measurement of the central angle and the measurement of the minor arc it intercepts.

Q2 Write a conjecture about the measurement of the central angle and the measurement of the major arc.

Q3 Write a conjecture about the measurements of an inscribed angle and the arc it intercepts.

Q4 Write a conjecture about all the inscribed angles that intercept the same arc.

Q5 Write a conjecture about angles inscribed in a semicircle.

Q6 Investigate conjecture Q3 to figure out why it must be true. Write your argument as clearly as you can, as if you were trying to convince someone who doesn't believe you.

EXPLORE MORE

EM1 In a new sketch, construct a circle and an arc on the circle. Measure the circumference of the circle, the arc angle, and the arc length.
Use the circumference and arc angle measurements to calculate an expression equal to the arc length. Explain what you did.

EM2 Use your conjecture in Q5 to come up with a method for constructing a right triangle. Describe your method.

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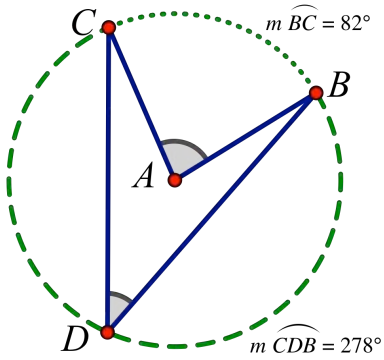

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$m\overset{\frown}{BC}$ measures the *arc angle* of $\overset{\frown}{BC}$.

$\overset{\frown}{CDB}$ is a *major* arc, because it is more than a semi-circle.

FORM AND TEST CONJECTURES

What conjectures can you make concerning the relationships between circles, their central angles, their inscribed angles, and their arcs?

Present each conjecture on a separate page of your sketch document. The page should include at least (1) a Hot Text statement of the conjecture and (2) a construction with an Animation or Movement button to illustrate the conjecture.

Choose a conjecture that you believe is true, and write an argument (in the form of a Hot Text caption) to explain why it must be true. You may need to construct additional objects to make your argument convincing, and Hide/Show buttons may help you to make your explanation clear.

EXPLORE MORE

Answer these two questions based on your conjectures, or on new conjectures that you create for the purpose.

How can you calculate the length of an arc if you know the arc angle? What other information do you need? Explain.

Invent at least two ways to construct a right triangle inscribed in a given circle. Explain why your constructions work.