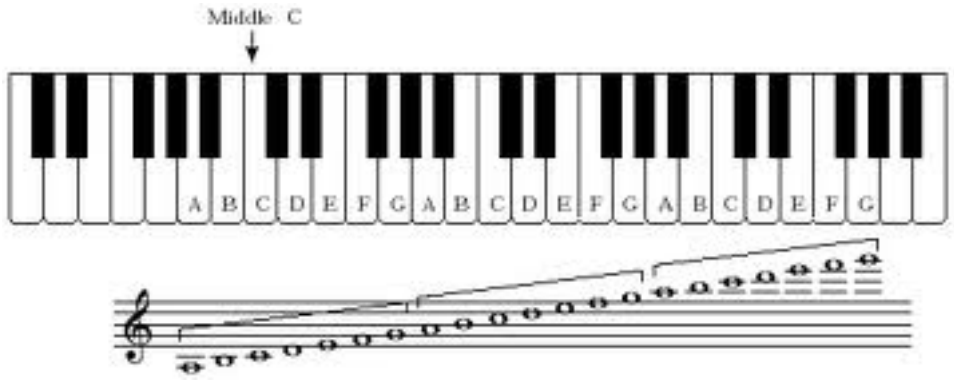


When two musical notes are an octave apart, their frequencies are in a ratio of 2/1. In other words, the higher note's frequency is 2 times that of the lower note.

1. The A above middle C (called A4) has a frequency of 440 per second. What is the frequency of A5, one octave higher than the A above middle C?

2. What is the frequency of A3? What's the frequency of A6? How about A0?

Two notes exactly an octave apart sound "good" together. In general, when two notes are played or sung together they sound good when the ratio of their frequencies is a simple ratio like 2/1, or 3/2, or 4/3, 5/4, or 6/5. (A “fifth” is 3/2, a “fourth” is 4/3, a “major third” is 5/4, and a “minor third” is 6/5.)

3. What are some other frequencies that would sound good with A4 (the A above middle C)? Find three examples, with at least two of the frequencies between A4 and A5.

4. Take the example of the note with a frequency 3/2 times the frequency of A4. By what fraction would you have to multiply that frequency to get to A5? Do you think these two notes would sound good together?

5. Notice that when you used 3/2 on A4, you had to use a different ratio to get to A5, the next octave. How could you use the same ratio twice to get from A4 to A5? What would be the frequency of this "middle" note, and what ratio did you use to get it? (This note divides the octave in half harmonically, so that step from A4 to your new note, and then from the new note to 880, sound very similar to the ear.)

6. You just figured out how to divide the octave into two steps that use the same ratio -- that is, two steps that sound the same harmonically. How could you divide the octave into three equal steps, so that all three steps sound the same?

7. Can you express the ratios you found in the last two problems as ratios of small integers? In other words, will these notes sound good together?

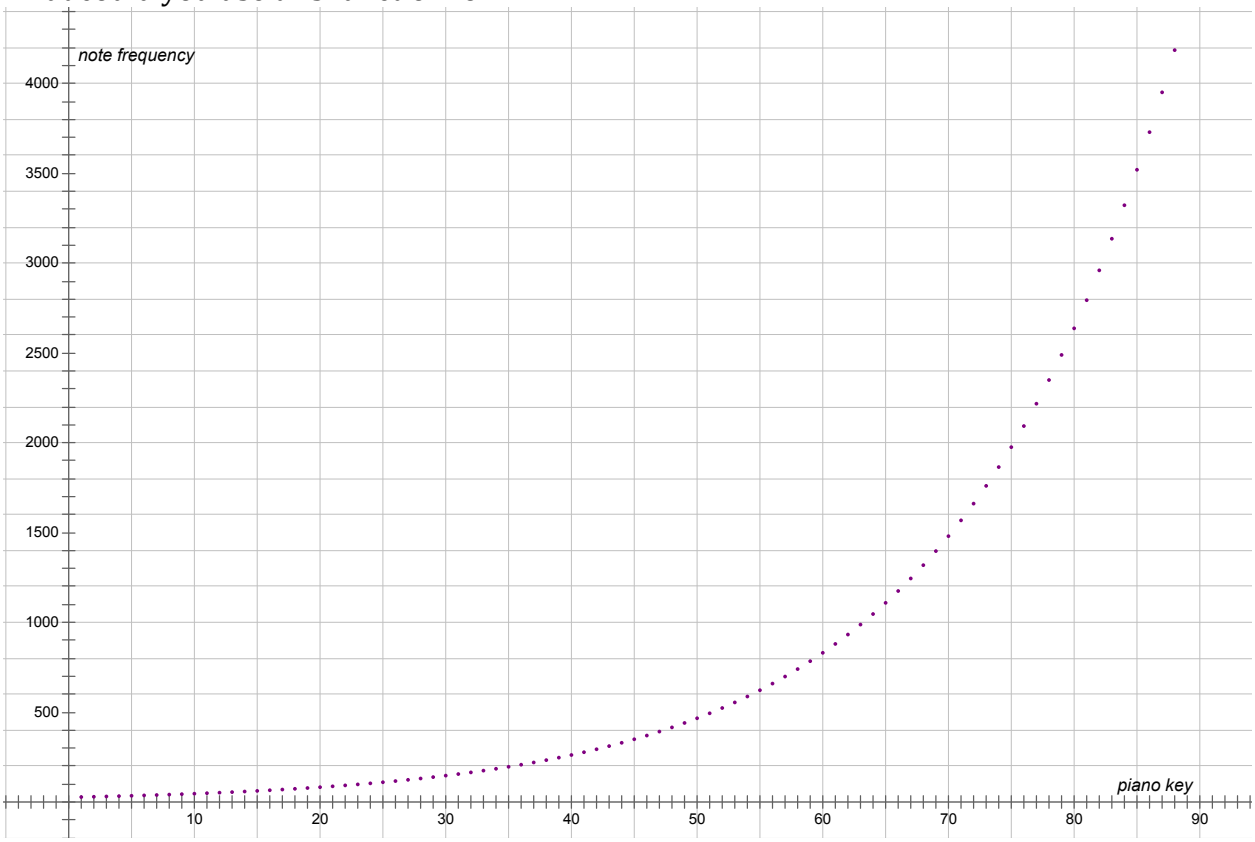
8. Try dividing the octave from A4 to A5 into four equal steps, or five, or six. Can you find some way to divide the octave that comes close to the frequencies that sound good – the frequencies at 3/2, or 4/3, or 5/4 times A4 (440 per second)?

9. Make a spreadsheet, or use Sketchpad, or use your graphing calculator to find a number of steps into which you can divide the octave and get really good results – results that are *very* close to the nice-sounding ratios.

10. Congratulate yourself if you’ve found a step size, and a number of steps, that comes within 1% of each of these important ratios, because you’ve just invented the even-tempered scale that’s the basis of all classical and most modern music!

Listen to the ratios here: http://www.musicalintervalstutor.info/listenpg.html

What could you use this function for?



And what could you use this function for?